The purposes of this study were 1) to investigate the contribution of individual terms such as the swing leg joint torques, motion-dependent term (MDT) and other terms to the generation of the knee joint angular velocity, and 2) to quantify the functional roles of the joint torques in the swing leg motion with consideration of the generating factor of the MDT. The swing leg was modeled as a linked system of three rigid segments in sagittal plane. Dynamic contributions and generating factors of the MDT were calculated using the equation of motion for the swing leg model. The results obtained in this study indicate that 1) the MDT was one of the great contributors to the generation of the knee joint angular velocity, and 2) the main generating factors of the MDT were the swing leg joint torques.

KEY WORDS: multi-body dynamics, knee joint angular velocity, motion-dependent term (MDT)

INTRODUCTION: Since the role of the support leg is to obtain propulsive forces from the ground and the role of the swing leg is to control step frequency and step length in sprinting, the swing leg motion is one of the determinative factors of performance at the maximal speed. Vardaxis and Hoshizaki (1989) showed that advanced sprinters produce higher peak joint torque powers than intermediate sprinters at early swing phase. They also reported that joint powers are generated and controlled by the hip and knee joints, respectively. Novacheck (1998) reported that the rectus femoris contracts eccentrically in early swing phase to restrain excessive knee flexion, and hamstrings contract eccentrically in late swing phase to restrain excessive knee extension and control momentum of the shank. That is, the muscles crossing the knee joint control swing leg motion through the absorption of the power. Schache et al. (2011) reported that the biomechanical loads of the hip extensor and knee flexor muscles during terminal swing phase become significantly high when running speed progresses toward maximal sprinting. The swing leg has large angular velocities at the hip and knee joints in the sprinting motion. For the high-speed swing motion, the motion-dependent term (MDT), which is consisted of product sum of the angular velocities of individual segments, would be a great contributor to the generation of the distal segment’s speed from the analysis based on the equation of motion for the linked multi-segment swing leg model. The generating factors of the MDT were quantified by Koike and Harada (2014) for a upper body model in tennis serve motion. This study converts the MDT into other input terms such as joint torques, gravity, and torso joint force.

The purposes of this study were 1) to investigate the contribution of individual terms such as the swing leg joint torques, MDT and other terms to the generation of the knee joint angular velocity, and 2) to quantify the functional roles of the joint torques in the swing leg motion with consideration of the generating factor of the MDT.

METHODS: Five male sprinters (height: 1.74±0.03m, body mass: 66.6±4.7kg, personal best time in the 100m: 10.79±0.27s) from a university track team performed 60m maximal sprints on a straight track in this experiment. The participants were instructed to start with crouch start from the starting block. Video data were recorded using three high-speed digital cameras (EXILIM-EX-F1, CASIO) operating at 300fps. The trajectories were smoothed using Butterworth digital filter and interpolated into 1000Hz data using cubic spline function. Two dimensional kinematic and kinetic data of the participants sprinting around 50m were analyzed by using the trajectory of 25 landmarks of the body. Joint torques of the swing leg
were calculated via an inverse dynamics. These data were normalized by the time of the swing phase as 0-100%.
The swing leg was modeled as a linked system of three rigid segments, whose lengths were set as constant values, in sagittal plane. The proximal end point of the thigh segment is constrained to the hip joint position.

An analytical form of the equation of motion for the system can be expressed as follows:
\[
\dot{V} = A_{Ta}T_a + A_V + A_GG + A_\eta
\]  
(1)

where vector \( V \) is the generalized velocity vector consisting of the translational and rotational velocity vectors with respect to the cg of individual segments, \( A_{ri} \) is the coefficient matrix of joint torque, \( A_V \) is the vector of MDT, \( A_G \) is the coefficient matrix of gravitational acceleration vector \( G \), and \( A_\eta \) is the vector of hip joint positional constraint term. After time integration of Eq. (1), multiplying selective matrix \( S_q \) that transforms the generalized velocity vector \( V \) into evaluation value \( q_{eval} \) yields the following equations:
\[
q_{eval} = S_q \left( A_{Ta}T_a \right) dt + S_q \left( A_V \right) dt + S_q \left( A_GG \right) dt + S_q \left( A_\eta \right) dt + S_q V_{init}
\]  
(2a)

\[
q_{eval} = C_{Ta} + C_V + C_G + C_\eta + C_{vo}
\]  
(2b)

where \( V_{init} \) is the initial value of the generalized velocity vector, \( C_{Ta}, C_V, C_G, C_\eta \) and \( C_{vo} \) are the contributions of joint torque term, MDT, gravitational term, hip joint positional constraint term, and initial velocity term, respectively.

When the MDT \( A_V \) of Eq. (1) is rewritten as the product of coefficient matrix \( \bar{A}_V \) and the generalized velocity vector as
\[
A_V = \bar{A}_V V
\]  
(3)

Eq. (1) can be rewritten as the following form:
\[
\dot{V} = A_V + \bar{A}_V V, \quad A_V = A_{Ta}T_a + A_GG + A_\eta
\]  
(4)

The equation of motion for the system, eq. (4), was discretized as follows:
\[
\dot{V}(k) = A_V(k) + \bar{A}_V(k) V(k), \quad A_V(k) = A_{Ta}(k)T_a(k) + A_G(k)G + A_\eta(k)
\]  
(5)

where \( k \) is the time of discrete system. The generalized acceleration vector was expressed by difference approximation shown as
\[
\dot{V}(k) = \frac{V(k + 1) - V(k)}{\Delta t}
\]  
(6)

Combining Eqs. (5) and (6) yields a recurrence formula for the generalized velocity vector \( V \) as follows:
\[
V(k + 1) = \Delta t A_V(k) + \left( E + \Delta t \bar{A}_V(k) \right) V(k)
\]  
(7)

The contribution to the generation of evaluation values after converting the MDT can be expressed using the selective matrix \( S_q(k) \) as follows:
\[
q_{eval}(k) = S_q(k) V(k)
\]  
(8a)

\[
= \bar{C}_{Ta} + \bar{C}_G + \bar{C}_\eta + \bar{C}_{vo}
\]  
(8b)

Eqs. (5) and (7) provide us the information about contribution of the input terms (e.g., swing leg joint torques, gravity, and positional constraint of hip joint) to the generation of evaluation values such as knee joint angular velocity and foot cg’s speed.

RESULTS: Figure 1 shows an example of time curves of the swing leg joint torques. The hip joint exerted flexion torque during 5-50% time and then exerted extension torque until foot contact. The knee joint exerted extension torque during 10-50% time and then exerted flexion torque until foot contact. The ankle joint exerted small torques over the swing phase.

When the knee joint angular velocity of the swing leg was selected as one of the evaluation values, the dynamic contributions of the individual terms were quantified via eqs. (2a,b) as shown by Figure 2. The total of joint torque contributions generated knee joint flexion angular velocity over the swing phase, while the contribution of MDT generated knee joint extension angular velocity over the swing phase. The gravitational term and the hip joint positional constraint term were small contributors to the knee joint angular velocity.

Figure 3 shows the contributions of individual terms to the generation of the knee joint angular velocity with consideration of the generating factors of the MDT. The contributions of the
individual terms were calculated using eqs. (7) and (8b). The total of joint torque contributions generated knee joint angular velocity over the swing phase. The initial velocity term was negative contributor to the knee joint angular velocity. The time curves of the contributions of the gravitational term and the hip joint positional constraint term resembled the shapes of the curves of the contributions calculated without consideration of the generating factors of the MDT (Figure 2).

Figure 4 shows the contributions of individual joint torques to the knee joint angular velocity with consideration of the generating factors of the MDT. The hip joint torque contributed to the flexion angular velocity during 0-50% time and then contributed to the extension angular velocity toward the toe-off. The knee joint torque contributed to the flexion angular velocity during 0-20% time, contributed to the extension angular velocity during 20-70% time, and then contributed to the flexion angular velocity toward the toe-off. The ankle joint torque contributed to the flexion angular velocity during 0-60% time, and then contributed to the extension angular velocity toward the toe-off.

**DISCUSSION:** Previous studies on high-speed swing motions, such as baseball pitching and tennis serve, reported that players utilize the MDT to generate large speeds of balls and
investigation of the differences between angular torque showed large value at middle and end intermediate sprinters. The magnitude of the contribution of MDT to the knee joint angular velocity increased gradually toward foot contact (Figure 2). An investigation of the differences between contributions with and without consideration of the generating factors of the MDT shows that the generating factors of the MDT were almost the swing leg joint torques (Figures 3 and 4). The results from the dynamic contribution analysis with consideration of the generating factors of the MDT indicate that the knee joint angular velocity was generated by not only knee joint torque but also by hip and ankle joint torques (Figure 4).

Although the ankle joint torque showed small values over the swing phase (Figure 1), the contribution of the joint torque to the knee joint angular velocity showed large value (Figure 4) because the high-speed movement of the foot segment, which is connected to the shank segment via the ankle joint, causes large contribution of MDT due to dynamic coupling of multi-segment system. The ankle joint torque was necessary to control the orientation of the foot segment. It is, therefore, difficult to utilize the ankle joint torque in order to modify the knee joint angular motion.

In order to investigate the influence of the inertial properties of the foot segment on the contribution of the MDT to the knee joint angular velocity, swing leg joint torques and dynamic contributions of the joint torques were calculated using an inertial parameters of the foot segment set into one tenth in magnitude. The dynamic contributions of the ankle joint torque to the knee angular velocity decreased to one eighth in extension and one sixth in flexion in this case. This result indicates that the torque was necessary not to generate the knee joint angular velocity but to control the orientation of the foot segment.

**CONCLUSION:** This study quantified dynamic contributions of the swing leg joint torques and motion-dependent term to the generation of the knee joint angular velocity based on equation of motion for a linked three-rigid-segment swing leg system in sagittal plane. The results are summarized as follows:

1. The motion-dependent term, which contributed negatively at early and terminal swing phases, was one of the great contributors to the generation of the knee joint angular velocity.
2. The main generating factors of the motion-dependent term were the swing leg joint torques.
3. The hip and ankle joint torques contributed to the knee extension angular velocity at the terminal swing phase, while the knee joint torque contributed largely to the knee flexion angular velocity at the phase.
4. Although the contribution of the ankle joint torque showed large value at middle and terminal swing phases, it is difficult to utilize the ankle joint torque in order to modify the knee joint angular motion because the torque was necessary to control the orientation of the foot segment.

**REFERENCES:**